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Shutdown reactivity worth of reactor control rods by a subcritical multiplication method

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SHUTDOWN REACTIVITY WORTH OF REACTOR CONTROL RODS BY A SUBCRITICAL MULTIPLICATION METHOD

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ABSTRACT

Two modified subcritical multiplication methods for measuring the reactivity worth of control elements were developed and investigated. The first involved the calibration of a subcritical count interval by the asymptotic period method. The second involved the calibration of a subcritical count interval by a rod interchange with the reactor regulating rod which had previously been calibrated by the asymptotic period method.

The accuracy of these methods is relatively poor. This inaccuracy is due to shadowing effects and to the spatial harmonics which exist in the core. An investigation of the spatial harmonics to determine the best location for the neutron detector is needed.

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I. INTRODUCTION

The measurement of reactivity [1,2,3] is one of the most important measurements in reactor physics. It is essential to reactor safety, design, operation, and experimental research.

In any nuclear reactor, controlled by rods of some neutron poison, the total reactivity worth of the control rods must be larger than the excess reactivity [4] built into the core, to insure that the reactor can be shut down. The smaller the amount of excess reactivity built into the core, the safer the core is. This excess reactivity, however, must be large enough to compensate for fission product poisoning, fuel burnup, and negative temperature coefficient, and to allow for efficient operation of the reactor. The excess reactivity of the University of Missouri at Rolla Reactor (UMRR) is less than one dollar, which insures that the reactor can not go prompt critical if the control rods are accidentally pulled from the core. In order for these conditions to exist in the core, an accurate determination of the reactivity worth of the control rods is needed.

The standard method of control rod worth determination is the asymptotic period method.* This method can easily

*Discussed in the Literature Survey.

be done on the regulating rod of the UMRR because its reactivity worth is less than the excess reactivity of the core. The shim-safety rods, however, have a much greater reactivity worth and therefore their worth cannot be determined unless the core is loaded to a higher excess reactivity. This is a time consuming and dangerous procedure and is done under special regulations put forth by the U. S. Atomic Energy Commission. Also, since rod reactivity worths change with the configuration of the fuel surrounding it, the rod worths determined are not exactly correct for the core configuration for which they are desired to be known. Also, it is impossible to determine the shutdown reactivity [4] of the core by this method.

The subcritical multiplication method* has long been recognized as a possible method for determining control rod reactivity worths. It requires no excess loading and is therefore less dangerous and time consuming. It can also be easily used to determine shutdown reactivity. The major drawback of this method is caused by harmonic effects.

In this work, a modified subcritical multiplication method has been designed and examined with the hope of retaining the advantages of the subcritical multiplication method while eliminating its liabilities.

*Discussed in the Literature Survey.

II. LITERATURE SURVEY

A. Methods of Reactivity Determination

There are two basic experimental techniques for fundamental reactor physics measurements: kinetic and static. Static methods are used to determine time independent core characteristics, but cannot be used to determine most dynamic characteristics. Kinetic methods yield precise values of the strictly dynamic parameters as well as many of the static parameters normally determined by exponential column experiments.

There are three basic types of kinetic techniques. These are asymptotic period measurements, reactivity perturbation techniques, and source perturbation techniques.

The asymptotic period measurement method is the most frequently used method and will be discussed in section B.

There are two main reactivity perturbation techniques: the rod-drop method and the rod oscillator method.

The rod-drop method [5,6,7,8,9] is the second most frequently used method of reactivity determination. This method is based on the transient response of the reactor to a rapid time variation in the flux due to the dropping of the reactor control rods when the reactor is in a critical state. Instantaneous flux as a function of time is recorded and appears as in Figure 1.

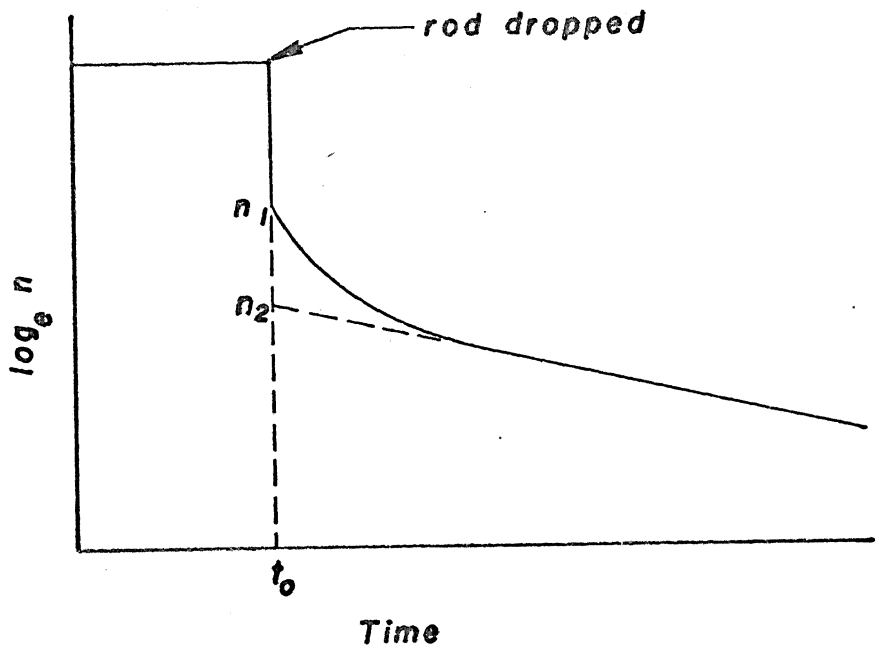


Fig. 1 Flux Level for the Rod Drop Method

The reactor kinetics equations are then used to determine the effective multiplication factor from the shape of the curve. The reactor kinetics equations are reduced to the following working equations.

$$k_{eff} = \frac{1 - n_o\beta/n_1}{1 - \beta}$$

$$k_{eff} = \frac{1 - n_o\beta_2/n_2}{1 - \beta_2}$$

β is the total delayed neutron fraction and β_2 is the delayed neutron fraction of the longest lived emitter.

The rod-drop method is advantageous because it requires no extra equipment and is very quick to perform. It can easily and safely measure large amounts of

reactivity. Also, it can be applied to hot reactors, since the initial critical power can overcome any existing extraneous source of neutrons such as photoneutrons (γ,n).

The rod-drop method has several disadvantages. The rods must be dropped from criticality, therefore limiting the rod combinations that can be measured. Also, the rod drop time is not instantaneous as is theoretically assumed, therefore limiting the accuracy of the method. The accuracy is poor if the graph is taken directly from the reactor $\log_e n$ chart; therefore, better recording equipment is required. A more accurate method is described by Moore[10].

The main difficulty, which is common to the kinetic methods, is the errors introduced by the harmonics. The harmonics are the deviations of the actual measured neutron distribution from the fundamental mode. The prompt harmonics, which are the deviation of the prompt neutron distribution from the prompt persisting mode, are the major contributor to the large systematic errors usually found in the rod-drop technique. In locations where the harmonics are positive (close to a dropped rod) the measured reactivity is larger than the true value. Far from these locations the situation is reversed.

The rod-oscillator method [11,12,13,14] is based on the oscillating component of the power level resulting from the periodic oscillation of a control rod. Functions which describe the behavior of the reactor power level for

a given type of reactivity variation can be derived from the reactor kinetics equations. For a variation of the form $\rho(t) = \delta e^{iwt}$, where w is a fixed frequency, the ratio of the maximum value of the oscillation component to the average power is directly proportional to the maximum value of reactivity.

$$\frac{n_{1\ max}}{n_0} = C\delta$$

δ is the rod reactivity worth and C is a constant. C can be experimentally determined by oscillating a known reactivity worth of the rod.

The rod-oscillator method has several distinct advantages. It is much quicker than other methods, when a large number of measurements are needed. This is true because the power level does not have to be exactly level since the oscillating component is the needed value rather than the power level itself. Also, there is no waiting for delayed neutron emitters to return to equilibrium because the power remains nearly constant. Since the rods are not moved, on the average, the harmonic effects are minimized in this method.

The major disadvantage of the method is that it requires a special piece of equipment [11] to perform the oscillation of the rods. This makes this method unfeasible unless reactivity measurement tests are to

be made quite regularly.

There are three major source perturbation techniques: source-jerk, "Rossi-α", and pulsed neutron.

The source-jerk method [6,11,15,16] is essentially the same as the rod-drop method, except that the neutron source is removed instead of inserting a control rod. The source-jerk method is also different in that it starts with the reactor in a subcritical state. If n_0 is the neutron level with the source in place and n_1 is the neutron level immediately after the source jerk (a few prompt neutron lifetimes), the subcritical reactivity of the core is as follows:

$$-\rho = \beta\left(\frac{n_0 - n_1}{n_1}\right)$$

Since a source is much smaller and lighter than a control rod, it is much easier to quickly remove from the core. The harmonic effects, which are a great problem in the rod-drop method, do not pose serious problems in the source-jerk method, since the flux shape in the core remains essentially unchanged.

The major disadvantage of the source-jerk method is caused by operation in the subcritical region. This requires a strong source (approximately ten curies) [15] to provide enough counts to prevent excessive data scatter. The Pu-Be source used with the UMRR has a strength of

approximately five curies [17] making the feasibility of using this method on the UMRR questionable. Another disadvantage is the necessity of a mechanism to jerk the source from the core.

The "Rossi-α" method [18,19] consists of the observation of the decay of individual neutron fission chains. This process is continued until enough chains are observed to obtain a statistically reliable measure of α (the prompt decay constant). The effective multiplication factor, k_{eff}, which in turn gives the reactivity, is determined from the following equations.

$$\alpha \equiv \frac{1}{n}\frac{dn}{dt} = \frac{k_p - 1}{L}$$

$$k_p = k_{eff}(1 - \bar{\gamma}\beta)$$

$\bar{\gamma}\beta$ is the effective delayed neutron fraction, L the prompt neutron lifetime, and k_p the prompt multiplication factor.

This method is limited to fast and intermediate systems in the neighborhood of delayed critical because of chain overlapping and background source problems for slow or highly subcritical systems. The "Rossi-α" experiment requires an excessive amount of time for data accumulation for L greater than 100 μsec. [18] (L = 500 μsec. for the UMRR).

The main advantage of this method is the high degree of accuracy which can be achieved.

This method requires a fast response, sensitive neutron detector and a multichannel analyzer with 0.25 or 0.50 μsec. width channels to display prompt neutron population vs. time.

The pulsed neutron method [6,20,21] consists of observing the transient behavior of the flux following a pulse of neutrons into the core. As in the "Rossi-α" method, the prompt decay constant α is measured.

$$\alpha_o \equiv \frac{1}{n}\frac{dn}{dt} = \frac{\bar{\gamma}\beta}{\Lambda}(\rho(\$) - 1)$$

α_o is the fundamental-mode decay constant, $\bar{\gamma}\beta$ the effective delayed neutron fraction, Λ the prompt-neutron generation time, and $\rho(\$)$ the reactivity in dollars. At delayed critical, $\rho(\$) = 0$ and

$$\alpha_{DC} = -\frac{\bar{\gamma}\beta}{\Lambda}.$$

α_{DC} is the decay constant at delayed critical. Thus, the fundamental-mode decay constant is given by

$$\alpha_o = \alpha_{DC}(1 - \rho(\$)).$$

The pulsed neutron method is a quick and straightforward method and has been shown to be in good agreement with the rod-drop and asymptotic period methods for reactivities as low as one dollar subcritical [20]. For reactivities down to approximately five dollars subcritical the pulsed neutron method has been found to be definitely

superior. Also, the pulsed neutron method provides its own reactivity calibration (α_{DC}). In the pulsed neutron method large attenuation of the prompt harmonics (major source of error in the rod-drop and source-jerk methods) is obtained by waiting for their decay.

The major disadvantage of the pulsed neutron method is the necessity of a pulse source of neutrons, which is generally supplied by the interaction of positive ions, from an accelerator, on a deuterium or tritium target. The effect of delayed harmonics prevents the use of this method below reactivities of approximately 10 to 15 dollars subcritical.[20]

There are three basic types of static techniques for reactivity measurement. They are neutron multiplication measurements, criticality determinations, and fuel substitution methods.

The criticality determination method [22] takes advantage of the settling out time for the reactivity (time for the period to return to infinity) when the reactor is brought to delayed critical. When a reactor is started up the precursor concentration exhibits a simple exponential buildup,

$$C_i(t) = C_{io} (1 - e^{-\lambda i t}).$$

The precursor decay rate is less than the precursor formation rate resulting in a time dependent deficit

in the neutron balance. This deficit settles out after the desired power level has been reached. The change in reactivity required to keep the power constant, as derived from the reactor kinetics equations, is

$$\frac{\rho}{\beta} = \frac{\sum_i \lambda_i C_{io} e^{-\lambda_i t}}{\sum_i \lambda_i C_{io}} .$$

C_i is the i^{th} precursor concentration, and λ_i is the i^{th} precursor's decay constant.

Only specific parts of the control rods can be calibrated because criticality must be maintained. Also, the core must be cold before starting the run. This method appears to have only limited use for reactivity calibrations.

The fuel-poison substitution method [23,24,25] involves the observation of the change in the control rod position, to retain the delayed critical condition, when a known amount of poison (neutron absorber) has been substituted for fuel in the core. The reactivity change can be determined using perturbation techniques. If the scattering cross section of the fuel and the poison are approximately the same, the reactivity change is,

$$\frac{\delta k}{k} = \frac{\int_{pv}\int_{e} \nu\Sigma_f \phi \overline{\chi \phi_o^{+}} \, dE dV}{\int_{cv}\int_{e} \nu\Sigma_f \phi \overline{\chi \phi_o^{+}} \, dE dV}$$

where PV is the poison volume, CV the core volume, χ the

neutron energy spectrum parameter, ϕ_o, $^+$ the adjoint flux, and the bars indicate an average over energy. This equation can be evaluated to sufficient accuracy by a multigroup, few-region computation.

An approximate method, for small reactivity changes, assumes the added poison affects the multiplication only through thermal utilization. Thus,

$$\frac{\Delta k}{k} = \frac{\Delta f}{f} = - \frac{\Delta \Sigma_a}{\Sigma_a} .$$

The major advantage of this method is the accuracy achieved due to the elimination of harmonic effects by the careful distribution of the poison throughout the core.

The major disadvantage of this method is the great amount of time required to place the poison in the core. The poison can be in the form of foils, wires, or some absorber dissolved in the moderator or coolant.

B. Asymptotic Period Method

The asymptotic period method [26,27,28,29] is the method most frequently used to calibrate control rods. The reactivity of the system is related to the stable reactor period (time for power to change by the factor e) through the reactor kinetics equations, which are derived from the diffusion theory. The reactor kinetics equations are as follows:

$$\frac{dn}{dt} = \frac{\rho - \beta}{L^*} \; n + \sum_{i=1}^{6} \lambda_i C_i$$

$$\frac{dC_i}{dt} = \frac{\beta i}{L^*} \; n - \lambda_i C_i$$

where n = neutron density

ρ = reactivity

β = total delayed neutron fraction

β_i = delayed neutron fraction of the i^{th} delayed group

L^* = prompt neutron generation time

λ_i = decay constant of the i^{th} group

C_i = concentration of the i^{th} precursor

Since these equations are linear first order and the variables are separable, solutions of the following form can be assumed:

$$n(t) = n_o \, e^{tw}$$

$$C_i(t) = C_{io} \, e^{tw}$$

w is an undefined parameter with reciprocal time units. Substituting these equations back into the kinetics equations and solving for reactivity,

$$\rho = wL^* + \sum_{i=1}^{6} \frac{w\beta i}{w + \lambda_i}$$

Since this equation is seventh degree in w, the neutron density has the form

$$n(t) = A_o \, e^{tw_o} + A_1 \, e^{tw_1} + \cdots + A_6 \, e^{tw_6}$$

where the w's are the roots of the reactivity equation.

If the reactivity is positive, it is found that one root (w_o) is positive and the other six roots are negative. Thus, after a short time, the neutron density reduces to

$$n(t) = A_o \, e^{tw_o} \, .$$

By definition, the stable reactor period (T_p) is

$$T_p \equiv \frac{1}{w_o} \, .$$

Therefore,

$$n(t) = n_o \, e^{t/T_p} \, .$$

Replacing w in the reactivity equation by $1/T_p$,

$$\rho = \frac{L^*}{T_p} + \sum_{i=1}^{6} \frac{\beta_i}{1 + \lambda_i T_p} \, .$$

This is the general relation between the reactivity and the stable reactor period, which is so important to reactor physics. This equation can be simplified if some assumptions are made. Assuming one average group of delayed neutrons,

$$\rho = \frac{\beta}{\lambda T_p + 1} \, .$$

This assumption is accurate for reactivities of about 0.025 percent or less. For reactivities less than 0.06 percent (periods greater than 130 seconds)

$$\rho = \frac{\beta}{\lambda T_p} .$$

For periods less than 130 seconds, the general equation should be used.

Using the delayed neutron fraction and decay constants as determined for in-pile U^{235} by Hughes, et. al.,[30] and $L^* = 5.0 \times 10^{-5}$ [31] the general reactivity equation is

$$\rho = \frac{.00005}{T_p} + \frac{.00025}{1+.01244T_p} + \frac{.00165}{1+.03051T_p} + \frac{.00148}{1+.1114T_p} + \frac{.00298}{1+.3014T_p} + \frac{.00087}{1+1.136T_p} + \frac{.00032}{1+3.014T_p}$$

The above relation is plotted in Figure 2.

The control rods are calibrated by the following procedure. The reactor is brought to criticality with the rod to be calibrated fully inserted. The rod to be calibrated is withdrawn a small distance and the transients are allowed to die out. It requires approximately two minutes for the transients to die to within 1 percent of the asymptotic period for periods less than 300 seconds [32]. The stable period is then observed from the period meter or the linear power meter. The reactor is then returned to the original critical power by inserting one of the other control rods. The procedure is repeated until the whole rod is calibrated. The integral rod worth

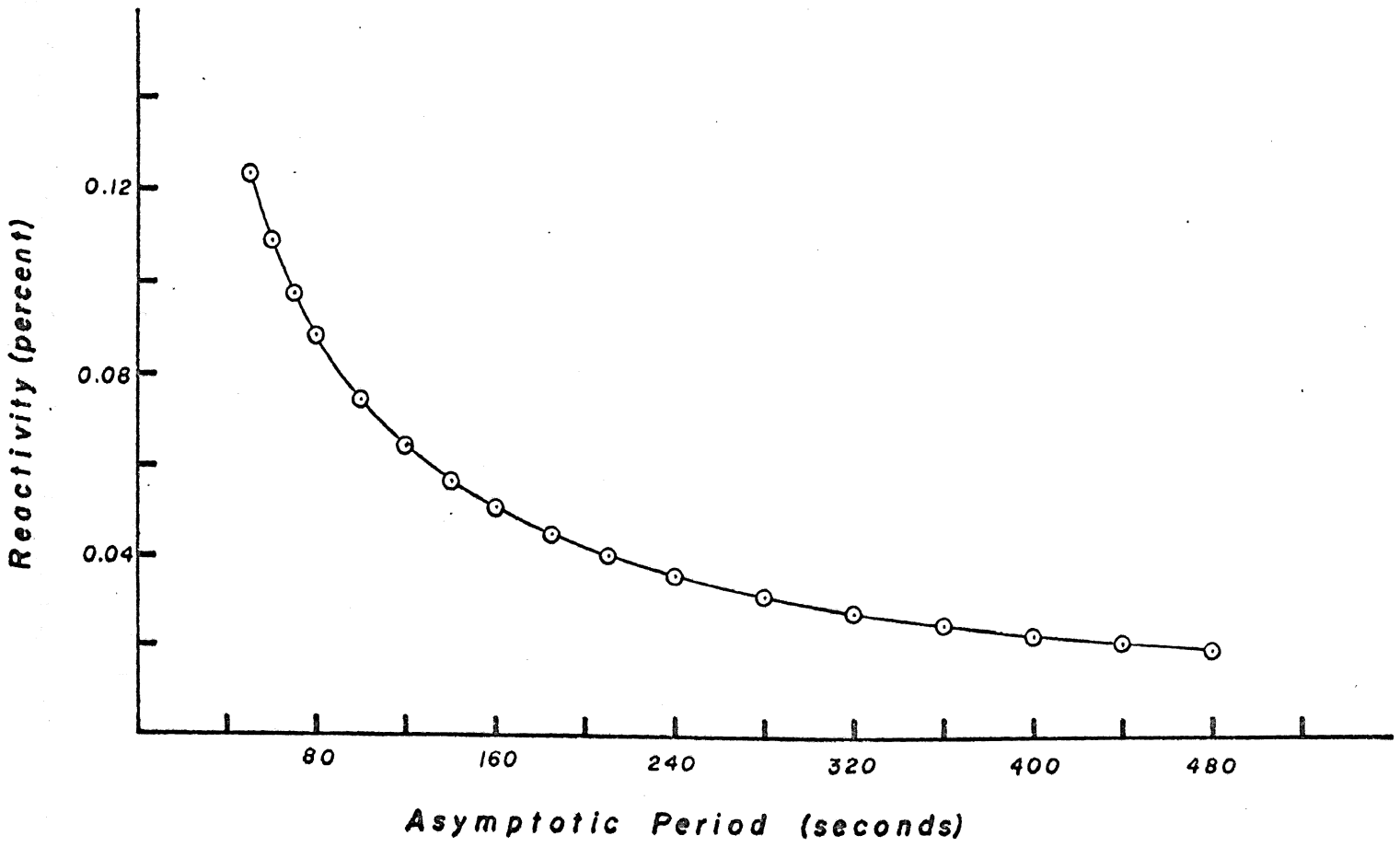


Fig. 2 The General Reactivity Relation for the the University of Missouri at Rolla Reactor

is determined by adding the individual reactivities.

One advantage of this method is that it requires no extra equipment. Another major advantage is that the higher-mode harmonics and the detector location have no effect since the higher harmonics are allowed to die away.

As noted in the Introduction, the asymptotic period method has several major disadvantages. The waiting time between period measurements should be approximately ten minutes for U^{235}. This is to allow the precursors to return to equilibrium. Period measurements taken before equilibrium is attained give values of reactivity which are too high. This error is on the conservative or safe side, however.

C. Subcritical Multiplication Method

The standard subcritical multiplication method of reactivity determination [33,34,35,36,37] is based on the approach to critical experiment.[38,39] The multiplication of a system is defined as the ratio of the total number of neutrons appearing in the system per source neutron. The total number of neutrons is the sum of the source neutrons and the neutrons from all the following fission generations. Therefore,

$$M = \frac{S + Sk + Sk^2 + \cdots}{S} = \frac{1}{1 - k}$$

where M is the multiplication, S the source strength, and k is the effective multiplication factor. Since the

count rate in the core is proportional to the neutron population, the multiplication can also be determined by the following equation:

$$M = \frac{C}{S_c}$$

M is the multiplication, C is the observed neutron count rate of the core, and S_c is the observed count rate of the source. Since reactivity (ρ) is defined as

$$\rho \equiv \frac{k - 1}{k}$$

the multiplication relates to the reactivity as follows:

$$M = \frac{\rho - 1}{\rho}$$

The source term is related to reactivity as follows:

$$S_c = \frac{C\rho}{\rho - 1}$$

The source term is found from the above equation by inserting a known amount of reactivity into the core which is critical. This known reactivity is usually the fine control rod, which has been calibrated by the asymptotic period method.

The above equation gives the reactivity in terms of the known source term when used in the following form.

$$\rho = \frac{S_C}{S_C - C}$$

By inserting the rods to be calibrated to different levels and observing the subcritical count rates, the reactivity worth of the rods can be determined. A wait of approximately ten minutes is required before each count is taken to insure that the delayed neutron precursors have died to an equilibrium concentration.(33)

The subcritical multiplication method is subject to several disturbing effects which cause higher harmonics. The major disturbing effects are due to core leakage, the spatial and spectral distribution of the source, and the location and spectral response of the detector.(33) Flux warpage and detector shadowing due to control rod location are also major causes of inaccuracy.(34) The harmonic effects disappear as delayed critical is approached. Therefore, the smaller the reactivity to be measured, the more accurate the measurement. R. M. Absalom, et. al., (40) have run several tests on rod location and interaction effects and on rod location and spectral response of the detector. They found that the apparant rod worth increases sharply when the rod is close to the detector. This is due to the local flux depression caused by the rod. They also found that reactivity values for a control rod measured at various detector positions varied as much as five percent due

to the geometrical dependence of the initial calibration of the fine control rod.

The ideal method to prevent inaccuracy due to flux warpage and detector shadowing would be to use a detector system of 4π geometry. This is not feasible to attain, however.

Rosenthal and Scicchitano [34] used the following method to find the best position to locate the detectors. They placed foils at two inch intervals around the core, at the same radius as the detector. The rods were inserted into the core just far enough to disturb the flux pattern. Thus, they obtained a peripheral flux map. They then placed the detectors at the positions at which the average flux was observed.

Bouzyk [41] recommends an investigation of the flux distributions at various reactor states. Then regions in which these distributions have similar shapes, independent of physical changes in the core, coincide with the regions of reduced abundance of spatial harmonics. From tests of this type, Bouzyk recommends that the following precautions be taken to minimize the effects of higher harmonics: central plane position of the source, location of the detector a few migration lengths from the perturbed region, and avoidance of mutual shadowing between source, detector, and control rods.

Cochran, et. al., [35] placed the source on the opposite side of the core from the detector and in a

central plane position when performing subcritical multiplication tests on the Bulk Shielding Reactor.

Another disadvantage of the subcritical multiplication method is the presence of photoneutrons (γ,n), after the reactor has been operated at a high power level. This requires a waiting period of from 12 to 24 hours after a high power run, i.e. 100 to 200 kw., to allow the gamma precursors to decay.

The subcritical multiplication method has several advantages. It is much safer and requires less time to run than the asymptotic period method. It has the capability to measure large amounts of reactivity in one measurement, such as the shutdown reactivity of a reactor. Reactivities determined in this manner are not a function of time and hence the inherent inaccuracies of a time dependent method are eliminated.

The agreement between the subcritical multiplication method and the asymptotic period method on critical experiment control system, on the MPR Zero Power Test Core, [34] was found to be better than 1 percent. This error represented the error incurred when rounding off figures for tabulation purposes.

The reproducibility of the subcritical multiplication method is also very good. Cochran, et. al., [35] found reactivities to be reproducible to about 17 parts in 10^4.

III. DISCUSSION

A. Theoretical

The multiplication in the core, when the rod being calibrated is in the i^{th} position, (M_i) is

$$M_i = \frac{C(k_i)}{C(o)} = \frac{1}{1 - k_i}$$

where k_i is the effective multiplication factor, when the rod is in the i^{th} position, C(o) is the count rate when there is no multiplying medium present, and $C(k_i)$ is the count rate when the rod is in the i^{th} position. Then,

$$M_i = \frac{C(k_i)}{C(k_o)} \cdot \frac{C(k_o)}{C(o)}$$

where k_o is the effective multiplication factor when the rod being calibrated is in the first position (completely inserted into the core). The multiplication at the first position can be defined as follows.

$$\frac{C(k_o)}{C(o)} = M_o = \text{constant}$$

Therefore,

$$M_i = \frac{C(k_i)}{C(k_o)} M_o$$

and

$$\frac{C(k_o)}{C(k_i)} = \frac{M_o}{M_i} = \frac{\frac{1}{1 - k_o}}{\frac{1}{1 - k_i}} = \frac{1 - k_i}{1 - k_o} .$$

The ratio of the count rates of the i^{th} and the i-1st positions of the rod is

$$\frac{C(k_{i-1})}{C(k_i)} = \frac{\frac{C(k_o)}{C(k_i)}}{\frac{C(k_o)}{C(k_{i-1})}} = \frac{\frac{1 - k_i}{1 - k_o}}{\frac{1 - k_{i-1}}{1 - k_o}} .$$

Therefore,

$$\frac{C(k_{i-1})}{C(k_i)} = \frac{1 - k_i}{1 - k_{i-1}} .$$

This equation can be expanded to the more general form,

$$\frac{C(k_i)}{C(k_N)} = \frac{1 - k_N}{1 - k_{i-1}}$$

where N is the N^{th} position of the rod. The reactor must not go critical at k_N or the N^{th} equation will be trivial because $C(k_N)$ would approach infinity. Either of the last two equations represents a system of N equations with N+1 unknowns; therefore, one known quantity is needed.

Another equation containing no new unknowns will make a system of N+1 equation and N+1 unknowns. This

equation can be obtained from the definition of reactivity,

$$\rho_i = \frac{k_i - 1}{k_i} .$$

$$\Delta\rho_{i-1\rightarrow i} = \frac{k_i - k_{i-1}}{k_i k_{i-1}}$$

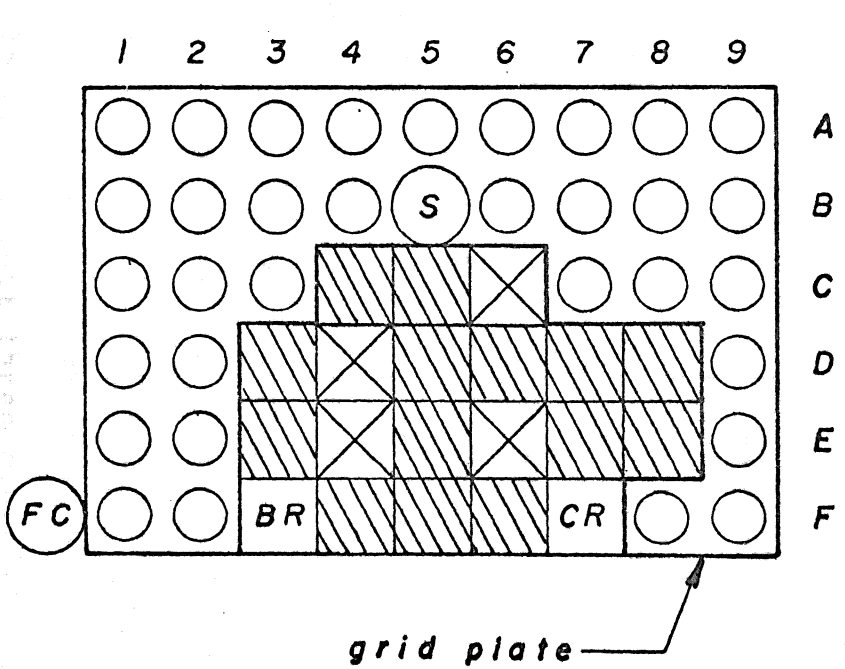
where $\Delta\rho_{i-1\rightarrow i}$ represents the change in reactivity of the core when the control rod being calibrated is moved from the i-1st position (delayed critical) to the i^{th} position. This is a positive amount of reactivity which can be measured by the asymptotic period method. An alternate method of measuring this amount of positive reactivity would be to return the reactor to delayed critical with the regulating rod, which had previously been calibrated by the asymptotic period method. By observing the positions of the regulating rod with the core at delayed critical and the control rod being calibrated at the i-1st and i^{th} positions, the reactivity worth of that interval can be found. The reactivity worth of any interval in which subcritical counts were taken at each end point can then be easily calculated.

B. Experimental

1. Equipment

These experiments were done on the UMRR. It is a swimming pool (modified BSR) type reactor, designed and built by Curtiss-Wright Corporation. The fuel

Core Loading 31T




- fuel element


- control rod

S - neutron source

FC - fission chamber

BR - core access element

CR - cadmium coated core access element

Fig. 3 The University of Missouri at Rolla Reactor

is highly enriched U^{235} and the moderator is light water. The core loading used, 31T, is shown in Figure 3. Core loading 32T was used in one series of experiments. It consists of core loading 31T plus a fuel element in core position C-7 and a half fuel element in position C-3.

The subcritical counts were taken from the scaler-timer in the reactor start-up channel which is shown in Figure 4.

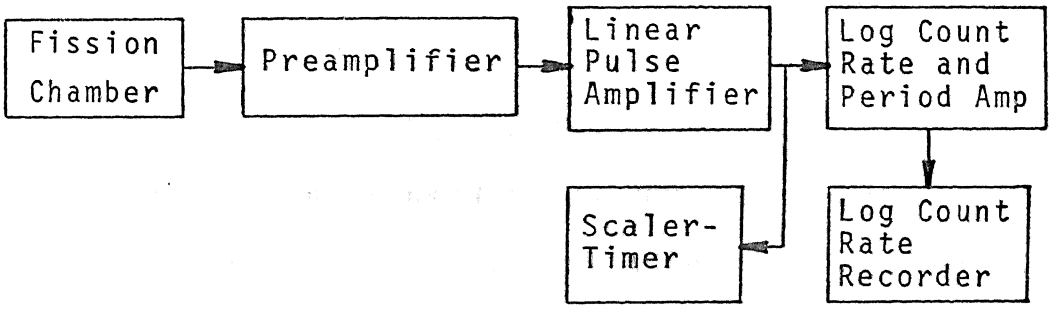


Fig. 4 The Start-up Channel

The detector used in these experiments is a Westinghouse type WL-6376 fission chamber. It is located in the core as shown in Figure 3. It can be moved vertically by the reactor operator and it is approximately 12 to 14 inches above the grid plate when in its lowest position. All tests were conducted with the fission chamber in this position. It is approximately 7 inches from the closest fuel element. This is almost three migration lengths. The fission chamber has a detection range of 1.4

to 1.4 X 10^5 neutrons/cm^2/second and a sensitivity of approximately 0.7 counts/neutron/cm^2, for thermal neutrons.

The preamplifier is a Honeywell Pulse Preamplifier, type 1906-(H1), which has a gain of 15. The amplifier is a Honeywell Linear Pulse Amplifier, type 1907-(J1), which is operated at a gain of over 12,000.

The scaler-timer is a Curtiss-Wright, model CW-220. It has the sensitivity to operate on pulses of from 2 to 4 volts with a duration of approximately 0.4 microseconds. The maximum count rate of the mechanical counter used is 25,000 counts per second.

All this equipment is built into the reactor control system making the use of extra equipment unnecessary.

2. Procedure

Two different procedures were used. The first involved a calibration of a small interval of the rod being calibrated by the asymptotic period method. The second method involved a calibration of a larger interval by a rod interchange at criticality with the regulating rod, which had been calibrated by the asymptotic period method.

The first method is as follows, using the calibration of control rod # 1 as an example.

1. Fully withdraw control rods # 2 and # 3.

2. Fully insert control rod # 1 and the regulating rod.
3. Withdraw control rod # 1 in intervals, taking subcritical counts at each interval, until the reactor becomes critical. Wait ten minutes at each interval before counting, to insure the decay of transients.
4. Bring the reactor to criticality, with control rod # 1 at some point where a subcritical count was taken, by withdrawing the regulating rod.
5. Determine the reactivity worth of one subcritical count interval by the asymptotic period method.
6. Determine the reactivity worth of all subcritical count intervals by the equations described in section III.A.

The second method is as follows, again using the calibration of control rod # 1 as an example.

1. Calibrate the regulating rod by the asymptotic period method.
2. Fully withdraw the regulating rod.
3. Gang withdraw rods # 1, # 2, and # 3 until the reactor is critical at 2 watts. Record the rod positions.

4. Fully withdraw rods # 2 and # 3 and insert rod # 1 till critical at 2 watts. Record the rod positions.
5. Insert the regulating rod to 10 inches. Withdraw rod # 1 until critical at 2 watts. Record rod positions. This rod interchange calibrates one subcritical interval.
6. Return rods # 2 and # 3 to the positions found in step 3. Fully withdraw the regulating rod.
7. Take subcritical counts with rod # 1 at the positions found in steps 4 and 5.
8. Take subcritical counts with rod # 1 in positions down to fully inserted.
9. Determine the reactivity worth of all subcritical count intervals by the equations described in section III.A.

These experiments were run with the source in core position B-5 and in position C-7. With the source in core position C-7, a major part of the fuel was between the source and the detector as prescribed by Bouzyk [41].

IV. DATA AND RESULTS

A. Data

The experimental data taken in this work is given in Appendix I.

B. Analytical Procedure

The solution of the pair of simultaneous equations

$$\Delta\rho_{i-1\to i} = \frac{k_i - k_{i-1}}{k_i k_{i-1}}$$

$$\frac{C_{i-1}}{C_i} = \frac{1 - k_i}{1 - k_{i-1}}$$

for k_i can be done by solving both equations for k_{i-1} and setting the results equal.

$$k_{i-1} = \frac{k_i}{1 + \Delta\rho k_i}$$

and

$$k_{i-1} = 1 - (1 - k_i)\frac{C_i}{C_{i-1}} .$$

Therefore,

$$\frac{k_i}{1 + \Delta\rho k_i} = 1 - (1 - k_i)\frac{C_i}{C_{i-1}} .$$

This can be expressed in the more general form,

$$\Delta\rho\frac{C_i}{C_{i-1}} k_i^2 + \left(\frac{C_i}{C_{i-1}} + \Delta\rho - \frac{C_i}{C_{i-1}}\Delta\rho - 1\right) k_i - \frac{C_i}{C_{i-1}} + 1 = 0 .$$

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The most common method used to solve this type of equation for k_i is by use of the quadratic equation. Therefore,

$$k_i = \frac{-B + (B^2 - 4AD)^{\frac{1}{2}}}{2A}$$

where

$$A = \Delta\rho \frac{C_i}{C_{i-1}}$$

$$B = \frac{C_i}{C_{i-1}} + \Delta\rho - \frac{C_i}{C_{i-1}} \Delta\rho - 1.0$$

$$D = - \frac{C_i}{C_{i-1}} + 1.0 .$$

From the known value, k_i, the total reactivity worth of the control rod can then be computed. This method was used, but it appeared to give inconsistent answers. Therefore, the value $\Delta\rho$, the known reactivity for the increment i-1 to i, was varied from $0.9\Delta\rho$ to $1.1\Delta\rho$ to see what effect an error in $\Delta\rho$ would have on the calculation of the total reactivity worth of the control rod. The results for several different known $\Delta\rho$'s are shown in Figure 5. The inconsistency was found to be due to the computer calculation of the square root term because $B^2 - 4AD$ was very close to zero (approximately 10^{-5} to 10^{-7}).

To avert this inconsistency another numerical method

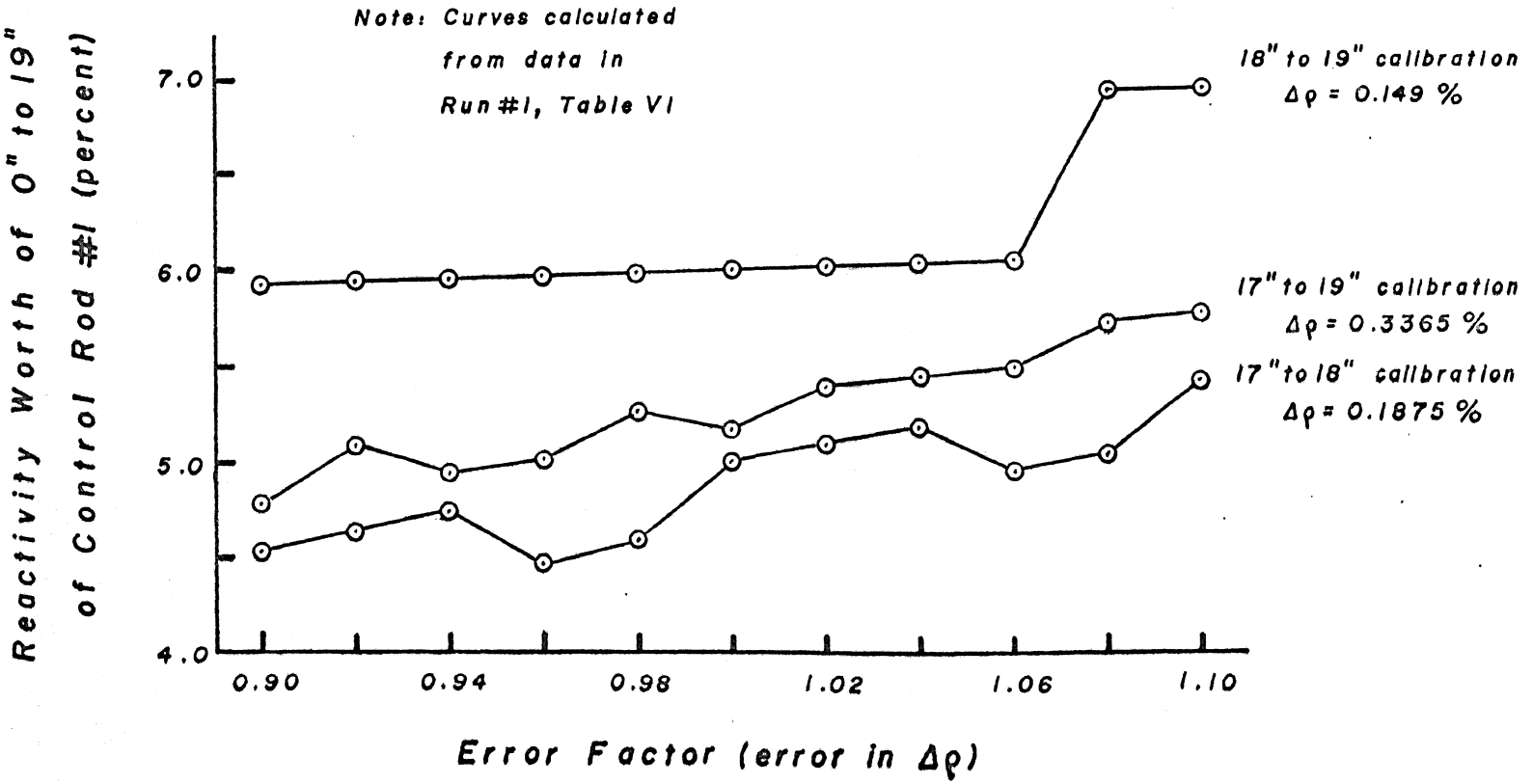

Fig.5 Effect of Error in Δρ on the Total Control Rod Worth When Solving by the Quadratic Equation

was tried. The quadratic equation can be written as

$$k_i = \frac{B}{2A}\left[-1 + \left(1 - \frac{4AD}{B^2}\right)^{\frac{1}{2}}\right] .$$

The square root term can then be replaced by the first five terms of a Maclaurin Series of the form $(1-x)^{\frac{1}{2}}$ where $x = 4AD/B^2$. The equation for k_i takes the form

$$k_i = -\frac{D}{B} - \frac{AD^2}{B3} - \frac{2A^2D^3}{B5} - \frac{5A^3D^4}{B7}$$

where A, B, and D are the same as above. Five terms of the series are sufficient to confine the error to less than 10^{-4} percent for values of k_i close to 1.0. The results of error in $\Delta\rho$, when using this equation, for the same known $\Delta\rho$'s used in Figure 5, are shown in Figure 6. It can be seen by comparing Figures 5 and 6 that care must be taken in the numerical analysis of the data.

The computer program used to determine the total reactivity worth of a control rod, using the above equation, is persented in Appendix II.

The data taken in these experiments only provides reactivity worths for portions of a control rod. It is desired to know the reactivity worth of the entire control rod. Therefore, an extrapolation of the results obtained for a portion of a control rod is needed. This

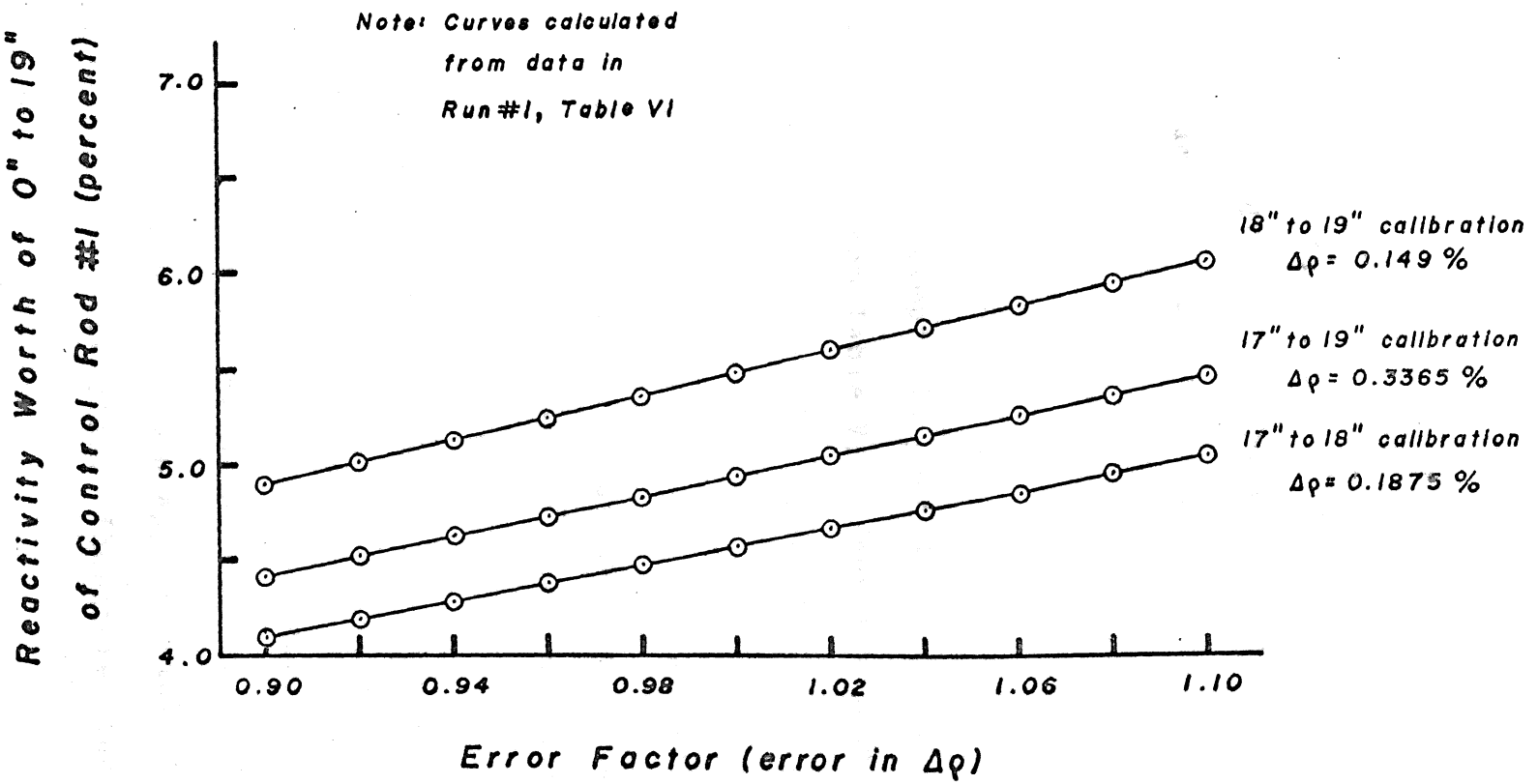

Fig.6 Effect of Error in Δρ on the Total Control Rod Worth When Solving with a Maclaurin Series

is done by fitting the results to a least squares fit[42] of the equation

$$\rho = A \cos^2 \frac{\pi x}{48}$$

where A is a constant and x is the rod position. This equation is predicted by perturbation theory. The computer program used to perform a least squares fit to this equation is described in Appendix III.

C. Results

The integral regulating rod reactivity worth, as measured by the asymptotic period method, is shown in Figure 7 (data from Table V). When the source is in core position B-5, there is apparently some shadowing effect which causes the integral reactivity curve to deviate from a general $\cos^2$ form in the vicinity of 17 to 20 inches withdrawn. The asymptotic period data was not least squares fit to a $\cos^2$ curve because the complete rod had been calibrated.

The results of the subcritical multiplication calibrations of control rods by method # 1 are given in Table I. The results from run # 3 appear to be about 20 percent lower than the other runs to calibrate control rod # 1. This run was taken approximately two hours after a 200 kw run on the reactor. Therefore, the reactor was

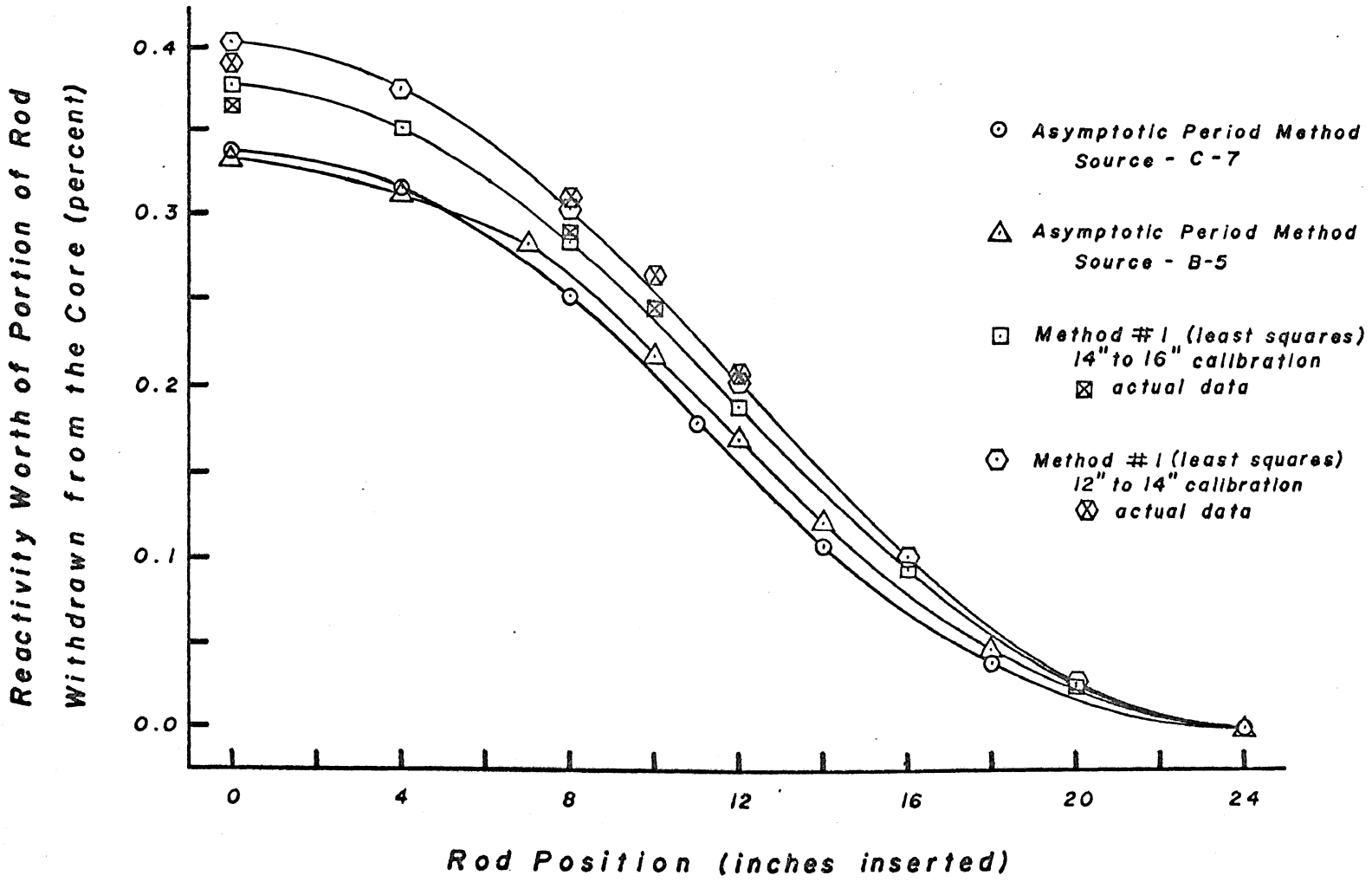

Fig. 7 Regulating Rod Worth

TABLE I

RESULTS OF SUBCRITICAL MULTIPLICATION CALIBRATION BY METHOD # 1 *

Run #	Rod or Rods Calibrated (Rod #)	Portion of Rod Calibrated (inches)	Asymptotic Period Calibration Interval (inches)	Total Rod Worth by Least Squares (percent reactivity)
1	1	0 - 19	17 - 18	3.4509 ± .0671
1	1	0 - 19	18 - 19	4.4586 ± .0851
2	1	0 - 23	20 - 22	3.6827 ± .1138
2	1	0 - 23	22 - 23	4.7330 ± .1444
2	1	0 - 23	20 - 23	3.7393 ± .1205
3	1	0 - 20	18 - 19	2.9551 ± .0320
3	1	0 - 20	19 - 20	3.9139 ± .0434
4	1	0 - 19.5	17.5 - 18.5	3.8858 ± .0616
4	1	0 - 19.5	18.5 - 19.5	4.9126 ± .0775
5	1	0 - 18.5	16.5 - 17.5	3.6119 ± .0613
5	1	0 - 18.5	17.5 - 18.5	4.4753 ± .0755
6	2	0 - 18	17 - 18	4.1016 ± .0552
7	3	0 - 19	18 - 19	5.0139 ± .0548

(continued on next page)

* Data from Table VI.

Table I (cont.)

Run #	Rod or Rods Calibrated (Rod #)	Portion of Rod Calibrated (inches)	Asymptotic Period Calibration Interval (inches)	Total Rod Worth by Least Squares (percent reactivity)
8	1, 2,& 3	6 - 24	22 - 23	10.0217 ± .0129
8	1, 2,& 3	6 - 24	23 - 24	9.3414 ± .0149
8	1, 2,& 3	6 - 24	22 - 24	8.3632 ± 0
9	1, 2,& 3	0 - 20.8	20.2 - 20.5	14.5840 ± .2837
9	1, 2,& 3	0 - 20.8	20.5 - 20.8	17.8557 ± .3954
9	1, 2,& 3	0 - 20.8	20.2 - 20.8	16.2223 ± .3785
10	Reg Rod	0 - 24	12 - 14	0.4035 ± .0099
10	Reg Rod	0 - 24	14 - 16	0.3768 ± .0092

still relatively hot. This is due to the (γ,n) reaction initiated by gamma rays from the decay of fission products. For this reason the results of run # 3 where not used in the final analysis. The average results for control rod # 1, as given in runs # 1, 2, 4, and 5, is 4.1056 ± 0.5408 percent reactivity. This is a deviation of approximately 13.2 percent.

The results of the subcritical multiplication calibration of control rods by method # 2 are given in Table II. The average results for control rod # 1, as given in runs # 1, 4, 7, and 10, is 3.9979 ± 0.5380 percent reactivity. This is a deviation of approximately 13.4 percent.

Combining these two sets of runs gives an average result of 4.0724 ± 0.5200 percent reactivity which has a deviation of 12.7 percent.

It can be seen from the results in Table I that, in general, the total rod worth for each run increases as the asymptotic period calibration interval approaches the end of the rod. This is true for all the runs but runs # 8 and # 10. These deviations appear to be due to two causes. First, the shadowing effect of the regulating rod, which is fully inserted during the process of taking the subcritical counts, causes the subcritical counts at the various rod positions to be low by a factor which decreases to zero as the rod approaches the fully

TABLE II

RESULTS OF SUBCRITICAL MULTIPLICATION CALIBRATION BY METHOD # 2 *

Run #	Rod or Rods Calibrated (Rod #)	Portion of Rod Calibrated (inches)	Asymptotic Period Calibration Interval (inches)	Total Rod Worth by Least Squares (percent reactivity)
1	1	0 - 17.72	16.00 - 17.72	3.5028 ± 0.0635
2	2	0 - 17.87	16.12 - 17.87	3.3107 ± 0.0596
3	3	0 - 18.70	17.20 - 18.70	3.8293 ± 0.1088
4	1	0 - 18.01	16.23 - 18.01	4.3331 ± 0.1148
5	2	0 - 18.16	16.38 - 18.16	4.8409 ± 0.1475
6	3	0 - 18.97	17.43 - 18.97	7.8064 ± 0.3119
7	1	0 - 17.75	16.02 - 17.75	3.5789 ± 0.0721
9	1, 2,& 3	0 - 20.89	19.815 - 20.89	28.0184 ± 2.4568
10	1	0 - 18.03	16.244 - 18.032	4.5770 + 0.1321

* Data from Table VII.

withdrawn position. Therefore, the subcritical counts taken for the interval 16 to 17 inches, for instance, deviates more below the true count rates than the subcritical counts taken for the interval 17 to 18 inches. The shadowing effect also causes deviation when the asymptotic period method is used to calibrate the subcritical count intervals. As the rod being calibrated is pulled further out of the reactor, the shadowing effect of the regulating rod becomes less. Therefore, the calibrated interval nearer the end of the control rod deviates below the true reactivity value less than the preceding interval. Since the total reactivity worth of the regulating rod is small compared to the control rods and it is at least three inches from the control rod, these shadowing effects are relatively small. The second and major cause for the deviations in the rod worths in the various runs in Table I was the harmonic effects introduced by the constant neutron flux source in core position B-5. These harmonics are negative near the rod being calibrated, and positive at positions further away from the rod. As the control rod is pulled from the core the reactor approaches criticality and the flux approaches the fundamental mode as the harmonics die away. The reactivity worth of the regulating rod, as calibrated in run # 10, decreases as

the asymptotic period calibration interval approaches the end of the rod. This is probably due to the harmonic effect caused by the large flux depression due to control rod # 2 which is fully inserted during this run.

An attempt was made in method # 2 to eliminate a large amount of the deviation displayed in method # 1. This was done by moving the source to core position C-7 where a much greater percent of the source neutrons must pass through the core before being seen by the detector. Also, the calibration interval was made larger so a more accurate calibration could be taken. Even though the calibration interval was made larger there is still a shadowing effect due to the regulating rod. This method prevents the measuring of more than one calibration interval, therefore preventing an analysis of this shadowing effect. The cause of the deviation in method # 2 for control rod # 1 appears to be due to the quantity of residual power remaining in the reactor when the runs # 1 and # 7 were made. This causes the reactor to go critical at a point where control rod # 1 is not pulled out as far as when the residual power has died away. Therefore, the results from runs # 4 and # 10 are preferred to runs # 1 and # 7. Runs # 5, 6, and 9 were made at the same times as runs # 4 and 10.

The least squares fit of the data seems to over

estimate the total reactivity worth of the control rod as shown in Figure 8. The error is approximately 6 percent. It can also be seen from the slope of the curve that the maximum differential rod worth occurs between 13 and 14 inches, and not at 12 inches as is assumed by using $\cos^2(\pi x/48)$ in the least squares fit.

Table III gives the results of the standard subcritical multiplication method. These values were calculated from the same data that was used for subcritical multiplication method # 2. They are appreciably different from the results of either subcritical method # 1 or # 2. The runs made using the 24 to 12 inch calibration interval give much larger answers than the runs using the 24 to 0 inch calibration interval. This deviation could be expected because the shadowing effect of the regulating rod when positioned at 12 inches should be quite large. This is especially true since the source is positioned right next to the regulating rod.

Table IV contains the results of the comparison tests of subcritical method # 1 and the asymptotic period method performed on core loading 32T. It is seen in Figure 9 that subcritical method # 1 underestimates the asymptotic period value of rod # 1 by approximately 7 percent. The calibration by subcritical method # 1 was made approximately a half hour after the rod was calibrated by the asymptotic period method. Therefore,

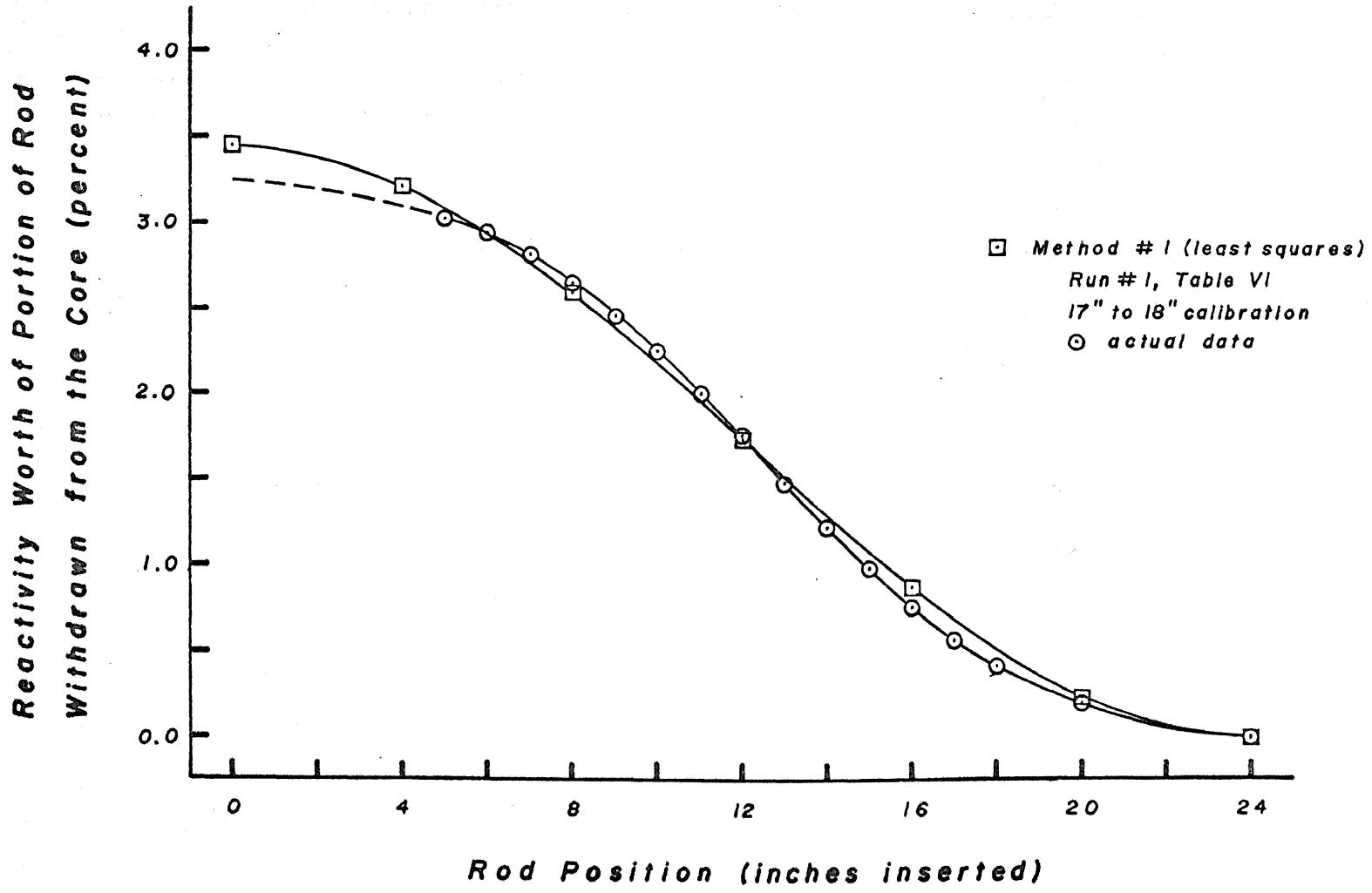

Fig. 8 Least Squares Fit to Subcritical Count Results

TABLE III

RESULTS OF STANDARD SUBCRITICAL MULTIPLICATION METHOD *

Run #	Rod or Rods Calibrated (Rod #)	Portion of Rod Calibrated (inches)	Regulating Rod Calibration Interval (inches)	Source Worth (S*)	Total Rod Worth By Least Squares (percent reactivity)
1	1	0 - 17.72	24 - 12	95.11	5.8194 ± 0.1936
1	1	0 - 17.72	24 - 0	72.40	4.3702 ± 0.1429
2	2	0 - 17.87	24 - 0	72.40	4.1560 ± 0.1221
3	3	0 - 18.70	24 - 0	72.40	4.1660 ± 0.1122
4	1	0 - 18.01	24 - 12	121.14	3.9750 ± 0.1702
4	1	0 - 18.01	24 - 0	70.60	2.2807 ± 0.0951
5	2	0 - 18.16	24 - 0	70.60	6.0214 ± 0.2768
6	3	0 - 18.97	24 - 0	70.60	8.0435 ± 0.3805
7	1	0 - 17.75	24 - 12	114.90	6.6110 ± 0.2594
7	1	0 - 17.75	24 - 0	78.50	4.4230 ± 0.1659
8	1, 2,& 3	0 - 18.00	24 - 12	114.90	27.4585 ± 0.8826
8	1, 2,& 3	0 - 18.00	24 - 0	78.50	17.3033 ± 0.4768

*Data from Table VII.

*$S = \frac{C\rho}{\rho - 1}$

TABLE IV

COMPARISON TESTS *

Test Method	Rod Calibrated (Rod #)	Portion of Rod Calibrated (inches)	Asymptotic Period Calibration Interval (inches)	Total Rod Worth (percent reactivity)	Total Rod Worth by Least Squares (percent reactivity)
Subcritical Method # 1	1	0 - 24	19 - 21	2.2712	2.2953 ± 0.0295
Subcritical Method # 1	1	0 - 24	21 - 24	2.2689	2.2930 ± 0.0293
Asymptotic Period	1	0 - 24	———	2.4794	2.6205 ± 0.0560

* Data from Table VIII.

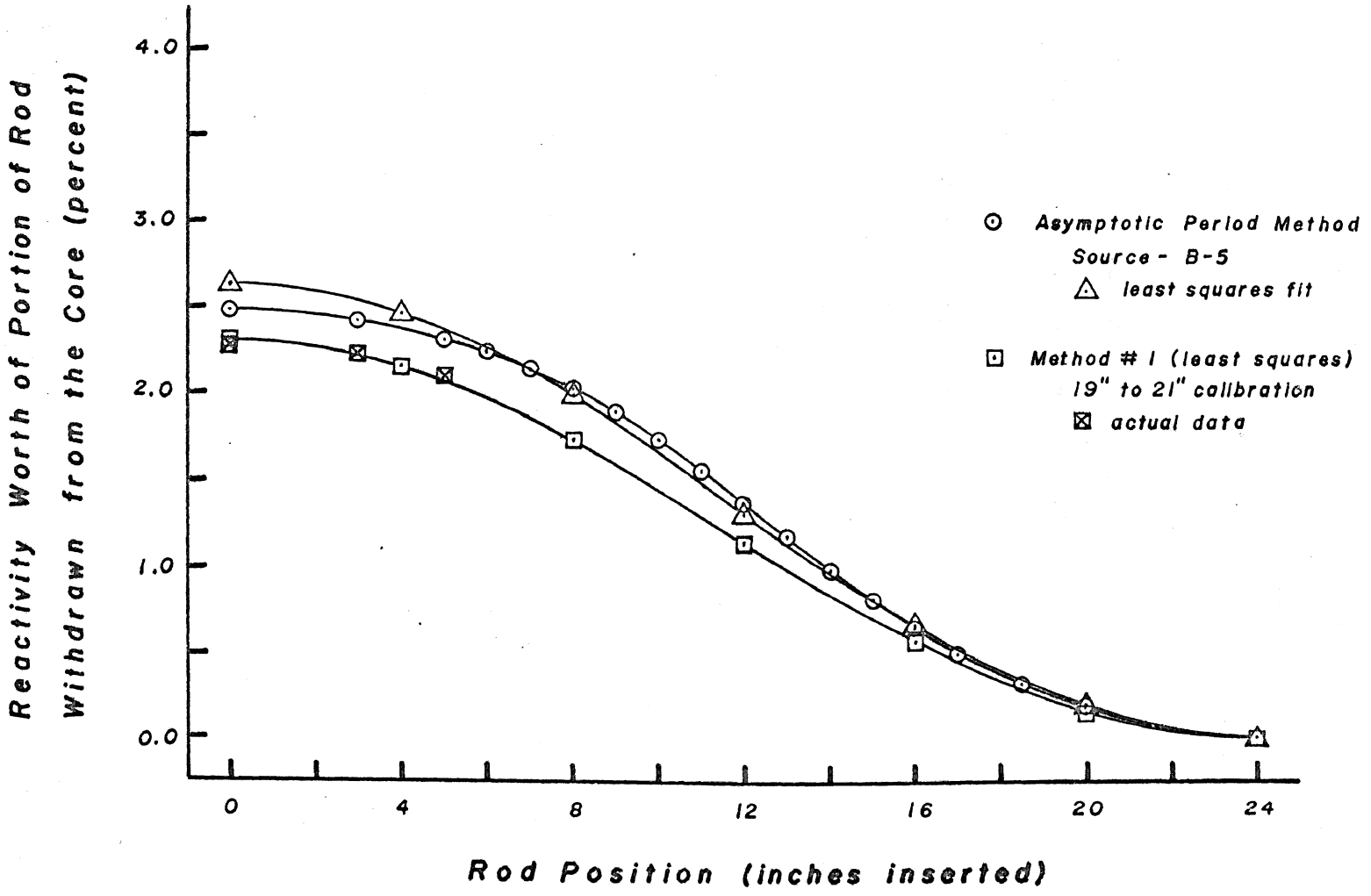

Fig. 9 Worth of Rod #1 in Core Loading 32T

the rod worth determined by subcritical method # 1 can be assumed to be several percent low, due to residual power, as was run # 3, by subcritical multiplication method # 1, on core loading 31T. Thus, the results of this comparison test are of little value to the experiment.

Correcting for the 6 percent error due to the least squares fit, the worth of rod # 1, as determined from runs # 4 and # 10 from Table II, is 4.1850 $\pm$ 0.1622 percent reactivity. This is an error of 3.9 percent. The rod worths of rods # 2 and # 3, determined from runs # 5 and # 6 in Table II, are also corrected in the same manner. The worth of rod # 2 is 4.6650 percent reactivity and the worth of rod # 3 is 7.5400 percent reactivity. The ganged worth of the shim-safety rods, found in run # 9 in Table II, appears to be unreasonably high. The ganged worth is probably between 10 and 15 percent as found by subcritical multiplication method # 1. This greater uncertainty is expected because of the appreciably greater harmonic effects at lower subcriticalities.

V. CONCLUSIONS

The two modified subcritical multiplication methods of determining reactivity worth, developed in this experiment, can be used to determine the total reactivity worth of a reactor control rod. These methods are quicker and safer than the commonly used asymptotic period method. They were performed using only equipment which was part of the reactor control system.

These two methods proved to be fairly inaccurate. The shadowing effect of the control rods contributed to this inaccuracy. However, the primary cause for inaccuracy of these methods was due to harmonics produced in the subcritical core. These two problems made the interpretation of the data extremely difficult. A great deal of care must be taken to see that the residual power of the reactor has died away or another element of inaccuracy becomes involved.

Method # 2 seemed to give more accurate results than method # 1. This was due to the placing of the neutron source on the opposite side of the core from the detector and to the larger calibration interval employed.

The inaccuracy of these methods made the determination of the shutdown reactivity of the reactor impossible.

VI. RECOMMENDATIONS

An investigation of the neutron flux distributions at various reactor subcritical states should be undertaken to determine the proper position to place the neutron detector to avoid the areas which contain an abundance of spatial harmonics. This will greatly improve the accuracy of the modified subcritical methods.

The accuracy of these methods would be greatly improved if they were performed on a more symmetrical core. Placing the control rods so that one is not directly adjacent to another will improve the error due to control rod shadowing.

An effort should be made to determine a function which more closely approximates the integral rod worth curve than does $\cos^2(\pi x/48)$.

The performance of subcritical method # 2 could be improved by inserting the full 24 inches of the regulating rod, instead of 14 inches (step 5), in the calibration of this method. This would eliminate some of the shadowing and harmonic induced error.

In the performance of subcritical method # 2, a waiting period should be included between steps 5 and 6. This will allow the residual power built up in steps 1 through 5 to die away before taking the subcritical counts.

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APPENDIX I

Experimental Data

The data for the calibration of the regulating rod by the asymptotic period method are given in Table V.

Table VI contains the data from the runs to determine the control rod reactivity worths by the first subcritical multiplication method set forth in section III.B.2. This includes the data for the asymptotic period calibration of one of the count intervals.

Table VII contains the data taken by the second subcritical multiplication method set forth in section III.B.2. Also included is the source calibration data so that the second subcritical multiplication method can be compared to the regular subcritical multiplication method discussed in section II.C.

Table VIII contains the data taken by the first subcritical multiplication method set forth in section III.B.2. for control rod # 1, for the core loading 32T. Also included are the asymptotic period data for control rod # 1. This set of data was taken so a direct comparison of the two methods could be made.

TABLE V

DATA FOR ASYMPTOTIC PERIOD CALIBRATION OF THE REGULATING ROD

Run # 1

Portion of Rod Measured (inches)	Doubling Time (seconds)	Period (seconds)
0 - 6	———	178.9
6 - 10	70.2	99.0
10 - 12	122.5	173.5
12 - 14	119.8	169.5
14 - 17	83.5	118.5
17 - 20	207.3	294.8
20 - 23.97	271.4	390.3

Source Position- B-5
Core Temperature- 116°F

Run # 2

Portion of Rod Measured (inches)	Doubling Time (seconds)	Period (seconds)
0.00 - 6.02	148.9	209.9
6.02 - 10.01	77.0	110.15
10.01 - 13.01	73.4	103.4
13.01 - 16.01	72.2	102.0
16.01 - 20.00	81.0	112.8
20.00 - 23.97	328.0	———

Source Position - C-7
Core Temperature - 85°F

TABLE VI

DATA FOR THE SUBCRITICAL MULTIPLICATION METHOD # 1 FOR CALIBRATING CONTROL RODS

Run # 1
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	1417
2	1446
4	1509
6	1634
7	1723
8	1865
9	2077
10	2331
11	2711
12	3257
13	3975
14	5187
15	7063
16	10509
17	17484
18	36587
19	136500

Rod # 2 & # 3 are fully withdrawn
Reg rod is fully inserted
Asymptotic period data
17" to 18": T_p=48.7, t_d=33.8
18" to 19": T_p=61.2, t_d=42.4
Core temperature - 112°F
Source position - B-5

Table VI (cont.)

Run # 2
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	5044.7
2	5129.4
4	5159.2
6	5580.0
8	6208.8
10	7547.8
12	9707.4
14	13874.6
16	22284.2
18	40410.6
20	85395.6
21	133032.2
22	207928.0
23	313306.0

Rods # 2 & # 3 are fully withdrawn
Reg rod is fully inserted
Asymptotic period data
20" to 22": T_p=49.8, t_d=36.2
22" to 23": t_d=160.0
20" to 23": T_p=35.4, t_d=24.8
Core Temperature - 118°F
Source position - B-5

Table VI (cont.)

Run # 3
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	453.6
18	5940.6
19	12934.0
20	37992.0

Rods # 2 & # 3 are fully withdrawn
Reg rod is fully inserted
Asymptotic period data
18" to 19": T_p=58.4, t_d=40.8
19" to 20": T_p=89.3, t_d=62.2
Core temperature - 116°F
This run was made two hours after a 200 kw run
Source position - B-5

Run # 4
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	4460.6
6	5214.7
17.5	65911.4
18.5	141067.3
19.5	467007.5

Rod # 2 & # 3 are fully withdrawn
Reg rod is fully inserted
Asymptotic period data
17.5" to 18.5": T_p=50.2, t_d=34.9
18.5" to 19.5": T_p=73.5, t_d=51.4
Core Temperature - 112°F
Source position - B-5

Table VI (cont.)

Run # 5
Control Rod # 1

Rod Position (inches)	Count Rate (count/minute)
0	735.54
6	863.80
16.5	8797.25
17.5	18917.30
18.5	77414.70

Rods # 2 & # 3 are fully withdrawn
Reg rod is fully inserted
Asymptotic period data
16.5" to 17.5": T_p=43.2, t_d=30.1
17.5" to 18.5": T_p=59.3, t_d=41.6
Core temperature - 102°F
Source position - B-5

Run # 6
Control Rod # 2

Rod Position (inches)	Count Rate (count/minute)
0	682.3
16	6049.0
17	11748.0
18	34399.6

Rods # 1 & # 3 are fully withdrawn
Reg rod is fully inserted
Asymptotic period data
17" to 18": T_p=46.35, t_d=32.4
Core temperature - 100°F
Source position - B-5

Table VI (cont.)

Run # 7
Control Rod # 3

Rod Position (inches)	Count Rate (counts/minute)
0	614.5
17	6539.7
18	13505.7
19	52660.2

Rods # 1 & # 2 are fully withdrawn
Reg rod is fully inserted
Asymptotic period data
18" to 19": T_p=39.0, t_d=26.93
Core Temperature - 100°F
Source position - B-5

Run # 8
All 3 Shim Rods (ganged)

Rod Position (inches)	Count Rate (counts/minute)
6	5480.8
22	185055.0
23	370266.0
24	738934.0

Reg rod is fully inserted
Asymptotic period data
22" to 23": T_p=53.6, t_d=36.0
23" to 24": T_p=145.9, t_d=99.1
22" to 24": T_p=37.5, t_d=25.9
Core temperature - 118°F
Source position - B-5

Table VI (cont.)

Run # 9
All 3 Shim Rods (ganged)

Rod Position (inches)	Count Rate (counts/minute)
0	175.6
12	375.8
19.8	7842.0
20.2	12621.0
20.5	20503.0
20.8	38279.0

Reg rod is fully inserted
Asymptotic period data
20.2" to 20.5": T_p=117.9, t_d=81.9
20.5" to 20.8": T_p=137.5, t_d=95.2
20.2" to 20.8": T_p=49.6, t_d=34.5
Core temperature - 100° F
Source Position - B-5

Run # 10
Regulating Rod

Rod Position (inches)	Count Rate (counts/minute)
0	7747.3
12	13100.3
14	16179.0
16	20283.3
24	35103.5

Rods # 1, # 2, and # 3 are at 20"
Asymptotic period data
12" to 14": T_p=148.0
14" to 16": T_p=179.0, t_d=125.6
Core temperature - 117°F
Source position - B-5

TABLE VII

DATA FOR THE SUBCRITICAL MULTIPLICATION METHOD # 2 FOR CALIBRATING CONTROL RODS

Run # 1
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	1978.8
8	2701.2
12	4921.0
14	8178.3
16	17258.7
17.72	55522.1

Rods # 2 & # 3 at 19.65"
Reg rod is fully withdrawn
Core temperature - 83°F
Source position - B-5
Calibration data
16" to 17.72" of rod # 1 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
Reg inserted to 12.00"
count - 57134.0
Reg inserted to 0.00"
count - 21645.0

Table VII (cont.)

Run # 2
Control Rod # 2

Rod Position (inches)	Count Rate (counts/minute)
0	2062.3
8	2775.3
12	4966.0
14	8219.0
16.12	18008.4
17.87	64233.3

Rods # 1 & # 3 at 19.65"
Reg rod is fully withdrawn
Core temperature - 83°F
Source position - B-5
Calibration data
16.12" to 17.87" of rod # 2 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
Same as in Run # 1

Table VII (cont.)

Run # 3
Control Rod # 3

Rod Position (inches)	Count Rate (counts/minute)
0	1992.8
8	2519.2
12	4190.8
14	6457.0
16	12915.7
17.20	23814.5
18.70	131246.0

Rods # 1 & # 2 at 19.65"
Reg rod is fully withdrawn
Core temperature - 83°F
Source position - B-5
Calibration data
    17.20" to 18.70" of rod # 3 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
    Same as in Run # 1

Table VII (cont.)

Run # 4
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	3663.5
8	5174.8
12	9733.0
14	16626.5
16.23	42038.6
18.01	167404.2

Rods # 2 & # 3 at 19.80"
Reg rod is fully withdrawn
Core temperature - 83°F
Source position - C-7, one foot above the grid plate
Calibration data
16.23" to 18.01" of rod # 1 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
Reg rod inserted to 12.00"
count - 65,849.3
Reg rod inserted to 0.00"
count - 21024.5

Table VII (cont.)

Run # 5
Control Rod # 2

Rod Position (inches)	Count Rate (counts/minute)
0	1437.1
8	1972.4
12	3827.2
14	6707.2
16.38	20229.8
18.16	102352.8

Rods # 1 & # 3 at 19.80"
Reg rod is fully withdrawn
Core temperature - 83°F
Source position - C-7, one foot above the grid plate
Calibration data
16.38" to 18.01" of rod # 2 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
Same as in Run # 4

≅ VII (cont.)

Run # 6
Control Rod # 3

Rod Position (inches)	Count Rate (counts/minute)
0	1079.6
8	1569.1
12	2916.0
15	6680.6
17.43	25844.0
18.97	213463.2

Rods # 1 & # 2 at 19.80"
Reg rod is fully withdrawn
Core temperature - 83°F
Source position - C-7, one foot above the grid plate
Calibration data
17.43" to 18.97" of rod # 3 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
Same as in Run # 4

Table VII (cont.)

Run # 7
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	2090.0
.8	2801.5
12	5092.0
14	8538.0
16.02	19040.8
17.75	64005.6

Rods # 2 & # 3 at 19.67"
Reg rod is fully withdrawn
Core temperature - 84°F
Source position - C-7, one foot above the grid plate
Calibration data
16.02" to 17.75" of rod # 1 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
Reg rod inserted to 12.00"
count - 62478.7
Reg rod inserted to 0.00"
count - 23389.6

Table VII (cont.)

Run # 8
All 3 Shim Rods (ganged)

Rod Position (inches)	Count Rate (counts/minute)
0	596.7
8	790.6
12	1344.6
14	2251.0
16	4443.6
17	7144.3
18	14490.5

Reg rod is fully withdrawn
Core temperature -84°F
Source position - C-7, one foot above the grid plate
Calibration data
Not taken
Source calibration data
Same as in Run # 7

Table VII (cont.)

Run # 9
All 3 Shim Rods (ganged)

Rod Position (inches)	Count Rate (counts/minute)
0	659.1
8	1020.1
12	1956.9
14	3108.8
16	5512.6
18	12574.3
19.815	46947.6
20.89	275328.4

Reg rod is fully inserted
Core temperature -84°F
Source position - C-7, one foot above the grid plate
Calibration data
19.815" to 20.89" of rods # 1, # 2, and # 3 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
Not taken

Table VII (cont.)

Run # 10
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	3454.8
8	4937.1
12	9305.5
14	16047.4
16.244	41756.4
18.032	168637.0

Rod # 2 is at 19.824"
Rod # 3 is at 19.808"
Reg rod is fully withdrawn
Core temperature - 84°F
Source position - C-7, one foot above the grid plate
Calibration data
16.244" to 18.032" of rod # 1 has reactivity worth equivalent to 10" to 24" of reg rod
Source calibration data
Not taken

TABLE VIII

DATA FOR CORE LOADING 32T

Data for the Subcritical Multiplication Method # 1
Control Rod # 1

Rod Position (inches)	Count Rate (counts/minute)
0	342.2
19	2215.2
21	3144.4
24	4270.0

Rod # 2 is fully withdrawn
Rod # 3 is fully inserted
Core temperature - 85°F
Source position - B-5
Asymptotic period data
19" to 21": T_p=58.8, t_d=41.6
21" to 24": T_p=109.7, t_d=77.4

Table VIII(cont.)

Asymptotic Period Data

Control Rod # 1

Portion of Rod Measured (inches)	Doubling Time (seconds)	Period (seconds)
0.0 - 2.0	75.7	106.2
2.0 - 4.0	47.1	65.4
4.0 - 5.5	31.1	44.2
5.5 - 7.0	20.6	29.1
7.0 - 8.0	24.9	35.2
8.0 - 8.5	61.3	86.8
8.5 - 9.0	56.0	78.8
9.0 - 9.5	54.5	76.4
9.5 - 10.0	48.9	69.3
10.0 - 10.5	48.9	69.0
10.5 - 11.0	50.0	71.1
11.0 - 11.5	47.7	67.1
11.5 - 12.0	50.9	71.5
12.0 - 12.5	50.0	70.5
12.5 - 13.0	56.0	79.1
13.0 - 14.0	22.0	31.2
14.0 - 15.0	25.5	36.1
15.0 - 16.0	29.8	42.1
16.0 - 17.0	39.4	55.5

(continued on next page)

Table VIII (cont.)

Asymptotic Period Data (cont.)

Control Rod # 1

Portion of Rod Measured (inches)	Doubling Time (seconds)	Period (seconds)
17.0 - 18.0	50.8	71.8
18.0 - 19.0	71.8	101.0
19.0 - 21.0	41.6	58.8
21.0 - 24.0	77.4	109.7

Source Position - B-5
Core Temperature - 85°F
Core Loading - 32T

APPENDIX II

Reactivity Determination Computer Program

The computer program used to calculate the reactivity worth of each control rod interval between positions where subcritical counts were taken is given in Table IX. This program contains the solution of the equations

$$\Delta\rho_{i-1\rightarrow i} = \frac{k_i - k_{i-1}}{k_i k_{i-1}}$$

and

$$\frac{C(k_i)}{C(k_{i-1})} = \frac{1 - k_{i-1}}{1 - k_i}$$

for k_i by using a Maclaurin series expansion as explained in section IV. Then the solution for the k for each subcritical count position is obtained by using the equation

$$\frac{C(k_i)}{C(k_N)} = \frac{1 - k_N}{1 - k_i}$$

which was derived in section III.A. The reactivity worth of each subcritical count interval is determined by the above equation for $\Delta\rho_{i-1\rightarrow i}$ and these incremental reactivities are summed to determine the total worth of the portion of the rod over which the subcritical counts were taken. A sample input data sheet is given in Table X and a sample computer output is given in table XI.

TABLE IX

COMPUTER PROGRAM TO DETERMINE REACTIVITY

```
/WAT4  NR120045,TIME=1,PAGES=5      CONNER L R  09/19/67
C
C  TOTAL SHIM ROD REACTIVITY WORTH BY A SUBCRITICAL
C  MULTIPLICATION METHOD
C
C     NN=NUMBER OF DATA SETS
C     N=NUMBER OF KNOWN P'S TO BE USED
C     M=NUMBER OF SUBCRITICAL COUNTS TAKEN/DATA SET
C     C=SUBCRITICAL COUNT
C     J=INTERVAL FOR WHICH P IS KNOWN
C     TJ=KNOWN STABLE REACTOR PERIOD
C     P=WORTH OF THE CALIBRATION INTERVAL
C     XK=EFFECTIVE MULTIPLICATION
C     STP=SUM OF THE INCREMENTAL REACTIVITIES
C     DP=INCREMENTAL REACTIVITIES
C     TP=TOTAL REACTIVITY OF THE PART OF THE ROD OVER
C     WHICH SUBCRITICAL COUNTS WERE TAKEN
C
      DIMENSION C(20),P(20),XK(20),R(20),DP(20)
      DO 100 NN=1,20
      READ (1,10)N, M
      READ (1,15) (C(I), I=1,M)
      DO 100 KK=1,N
      READ (1,10) J
      READ (1,15) T(J)
      P(J)=0.00005/TJ+0.00025/(1.+0.01244*TJ)+0.00165/1.+
     2.03051*TJ)+0.00148/(1.+0.1114*TJ)+0.00298/(1.+0.301
     .34*TJ)+0.00087/(1.+1.136*TJ)+0.00032/(1.+3.014*TJ)
      WRITE (3,20) P(J)
      L=J+1
      A=(P(J)*C(J+1))/C(J)
      B= C(J+1)/C(J) + P(J) - (P(J)*C(J+1))/C(J) - 1.0
      CC= -C(J+1)/C(J) + 1.0
      XK(J+1)=-CC/B-A*CC**2/B**3-2.0*A**2*CC**3/B**5-5.0
     2*A**3*CC**4/B**7
      DO 55 I=1,M
      IF (I-J-1) 40,55,40
   40 XK(I) = 1.0 - (1.0-XK(J+1))*C(J+1)/C(I)
   55 CONTINUE
      LK = M-1
      STP = 0.0
      DO 65 I=1,LK
   65 DP(I) = (XK(I+1)-XK(I))/(XK(I+1)*XK(I))
      DP(M) = (1.0-XK(LK))/XK(LK)
      WRITE (3,70)
      DO 66 LLL=1,M
      STP = STP + DP(LLL)
   66 WRITE (3,50) XK(LLL), DP(LLL), STP
```

```
      TP = (XK(M)-XK(1))/(XK(M)*XK(1))
      MM = M+2
  100 WRITE (3,60) TP
      CALL EXIT
   10 FORMAT (215)
   15 FORMAT (4E15.8)
   20 FORMAT ('   P(J) = ' E15.8/)
   50 FORMAT (5E17.8)
   60 FORMAT (' TOTAL REACTIVITY WORTH = ' E17.8///)
   70 FORMAT (9X,'K' 16X,'RHO' 14X, 'STP'/)
      END
```

TABLE X

SAMPLE INPUT DATA FOR THE REACTIVITY PROGRAM IN TABLE IX*

```
/DATA
    2   17
+0.14170000E+04+0.14460000E+04+0.15090000E+04+0.16340000E+04
+0.17230000E+04+0.18650000E+04+0.20770000E+94+0.23310000E+04
+0.27110000E+04+0.32570000E+04+0.39750000E+04+0.51870000E+04
+0.71630000E+04+0.10590000E+05+0.17480000E+05+0.36587000E+05
+0.13650000E+06
   16
+0.61200000E+02
   15
+0.48700000E+02
/END
```

*Data for Run # 1 in Table VI. Each line contains the data on one card.

TABLE XI

SAMPLE COMPUTER OUTPUT FOR THE INPUT DATA IN TABLE X

P(J) = 0.10748500E-02

K	RHO	STP
0.96212180E 00	0.82000570E-03	0.82000570E-03
0.96288150E 00	0.16687550E-02	0.24887600E-02
0.96443110E 00	0.20171870E-02	0.54059470E-02
0.96715210E 00	0.18107380E-02	0.72166840E-02
0.96884880E 00	0.25206510E-02	0.97373350E-02
0.97122070E 00	0.31047720E-02	0.12842100E-01
0.97415820E 00	0.29587270E-02	0.15800830E-01
0.97697410E 00	0.33703260E-02	0.19171150E-01
0.98020160E 00	0.34427360E-02	0.22613890E-01
0.98352060E 00	0.30679570E-02	0.25681840E-01
0.98649720E 00	0.32316770E-02	0.28913520E-01
0.98965230E 00	0.27984320E-02	0.31711950E-01
0.99240070E 00	0.25238620E-02	0.34235810E-01
0.99489260E 00	0.20542910E-02	0.36290100E-01
0.99693010E 00	0.16101260E-02	0.37900220E-01
0.99853300E 00	0.10757780E-02	0.38976000E-01
0.99960670E 00	0.14691440E-02	0.40445140E-01

TOTAL REACTIVITY WORTH = 0.38976020E-01

P(J) = 0.12584980E-02

K	RHO	STP
0.97038200E 00	0.63038520E-03	0.63038520E-03
0.97097600E 00	0.12836850E-02	0.19140710E-02
0.97218780E 00	0.22462070E-02	0.41602770E-02
0.97431540E 00	0.13956480E-02	0.55559240E-02
0.97564210E 00	0.19446610E-02	0.75005850E-02
0.97749670E 00	0.23982620E-02	0.98988450E-02
0.97979360E 00	0.22884040E-02	0.12187240E-01
0.98199540E 00	0.26104020E-02	0.14797650E-01
0.98451910E 00	0.26704030E-02	0.17468050E-01
0.98711430E 00	0.23830450E-02	0.19851090E-01
0.98944180E 00	0.25136990E-02	0.22364790E-01
0.99190890E 00	0.21795860E-02	0.24544370E-01
0.99405800E 00	0.19679210E-02	0.26512290E-01
0.99600640E 00	0.16034090E-02	0.28115700E-01
0.99759960E 00	0.12577650E-02	0.29373460E-01
0.99885290E 00	0.84087340E-03	0.30214330E-01
0.99969250E 00	0.11484080E-02	0.31362740E-01

TOTAL REACTIVITY WORTH = 0.30214360E-01

APPENDIX III

$\cos^2$ ($\pi X/48$) Least Squares Fit Computer Program

The computer program used to calculate the least squares fit of the reactivity versus rod position data obtained from the reactivity determination program is contained in Table XII. It fits the data to the equation,

$$\rho = A \cos^2 \frac{\pi x}{48}$$

where x is the control rod position, and A is a constant, for values of x from 0 to 24 inches. This program also calculates the standard deviation of the input data, from the least squares fit. A sample input data sheet is given in Table XIII, and a sample computer output is given in Table XIV.

TABLE XII

COMPUTER PROGRAM TO FIT REACTIVITY TO $\cos^2(\pi X/48)$ BY A LEAST SQUARES TECHNIQUE

```
/WAT4  NR120045,TIME=1,PAGES=5          CONNER L R  09/26/67
C
C   LEAST SQUARES FIT TO THE EQUATION RHO = A*COS(PI*X/48)
C
C      X=ROD POSITION
C      Y=INPUT REACTIVITY VALUE
C      RHO=LEAST SQUARES REACTIVITY VALUE
C      LRC=NUMBER OF DATA SETS
C      K=NUMBER OF POINTS/DATA SET
C      SD=STANDARD DEVIATION
C
       DIMENSION X(25),Y(25),RHO(25)
       RA(S)=(COS(PI*S/48))**2
       PI=3.1415927
       DO 50 LRC = 1,15
       READ (1,100) K
       DO 5 I=1,K
     5 READ (1,110) X(I),Y(I)
       TOP=0.0
       BOT=0.0
       DO 10 I=I,K
       R=X(I)
       XX=RA(R)
       TOP=TOP+Y(I)*XX
    10 BOT=BOT+XX**2
       A=TOP/BOT
       DO 20 I=1,25
       T=I-1
    20 RHO(I)=A*RA(T)
       DO 25 J=1,25
       JJ=J-1
    25 WRITE (3,120) JJ,RHO(J)
       TYS=0.0
       DO 30 I=1,K
    30 TYS=TYS+Y(I)**2
       SUM=TYS-2.0*A*TOP+A**2*BOT
       TT=K-1
       SD=SQRT(SUM/TT
       WRITE (3,130) SD
    50 CONTINUE
       CALL EXIT
   100 FORMAT (15)
   110 FORMAT (2E15.8)
   120 FORMAT (15,5X,E15.8)
   130 FORMAT (' STANDARD DEVIATION IS',5X,E15.8///)
       END
```

TABLE XIII

SAMPLE INPUT DATA FOR THE LEAST SQUARES FIT PROGRAM*

```
/DATA
   16
+0.22000000E+02+0.82000570E-03
+0.20000000E+02+0.24887600E-02
+0.18000000E+02+0.54059470E-02
+0.17000000E+02+0.72166840E-02
+0.16000000E+02+0.97373350E-02
+0.15000000E+02+0.12842100E-01
+0.14000000E+02+0.15800830E-01
+0.13000000E+02+0.19171150E-01
+0.12000000E+02+0.22613890E-01
+0.11000000E+02+0.25681840E-01
+0.10000000E+02+0.28913520E-01
+0.09000000E+02+0.31711950E-01
+0.08000000E+02+0.34235810E-01
+0.07000000E+02+0.36290100E-01
+0.06000000E+02+0.37900220E-01
+0.05000000E+02+0.38976000E-01
   16
+0.22000000E+02+0.63038520E-03
+0.20000000E+02+0.19140710E-02
+0.18000000E+02+0.41602770E-02
+0.17000000E+02+0.55559240E-02
+0.16000000E+02+0.75005850E-02
+0.15000000E+02+0.98988450E-02
+0.14000000E+02+0.12187240E-01
+0.13000000E+02+0.14797650E-01
+0.12000000E+02+0.17468050E-01
+0.11000000E+02+0.19851090E-01
+0.10000000E+02+0.22364790E-01
+0.09000000E+02+0.24544370E-01
+0.08000000E+02+0.26512290E-01
+0.07000000E+02+0.28115700E-01
+0.06000000E+02+0.29373460E-01
+0.05000000E+02+0.30214330E-01
/END
```

*Output data in Table XI.

TABLE XIV

SAMPLE COMPUTER OUTPUT FOR THE INPUT DATA IN TABLE XIII

0	0.44585960E-01
1	0.44395240E-01
2	0.43826340E-01
3	0.42889010E-01
4	0.41599270E-01
5	0.39979190E-01
6	0.38056500E-01
7	0.35864100E-01
8	0.33439470E-01
9	0.30824140E-01
10	0.28062840E-01
11	0.25202810E-01
12	0.22292990E-01
13	0.19383180E-01
14	0.16523140E-01
15	0.13761840E-01
16	0.11146490E-01
17	0.87218840E-01
18	0.65294720E-02
19	0.46067830E-02
20	0.29867080E-02
21	0.16969660E-02
22	0.75962580E-03
23	0.19072600E-03
24	0.43936540E-14

STANDARD DEVIATION IS 0.85143480E-03

0	0.34500490E-01
1	0.34361540E-01
2	0.33921540E-01
3	0.33196050E-01
4	0.32197790E-01
5	0.30943850E-01
6	0.29455690E-01
7	0.27758770E-01
8	0.25882120E-01
9	0.23857850E-01
10	0.21720610E-01
11	0.19506940E-01
12	0.17252750E-01
13	0.15002560E-01
14	0.12788890E-01
15	0.10651650E-01
16	0.86273770E-02

```
  17       0.67507280E-02
  18       0.50538070E-02
  19       0.35656460E-02
  20       0.23117090E-02
  21       0.13134500E-02
  22       0.58794950E-03
  23       0.14762170E-03
  24       0.34006860E-14
STANDARD DEVIATION IS      0.67064630E-03
```

VITA

The author was born on March 14, 1944, in Tulsa, Oklahoma. He received his primary and secondary education in the public schools of Bartlesville, Oklahoma. He received a Bachelor of Science Degree in Engineering Physics in June, 1966, from the University of Oklahoma.

He entered the Graduate School of the University of Missouri at Rolla in September, 1966, and has held a National Science Foundation Traineeship for the period September, 1966, to August, 1967.